Limit of Resolution

Resolution is the distance at which a lens can barely distinguish two separate objects.

Resolution is limited by aberrations and by diffraction. Aberrations can be minimized, but diffraction is unavoidable; it is due to the size of the lens compared to the wavelength of the light.

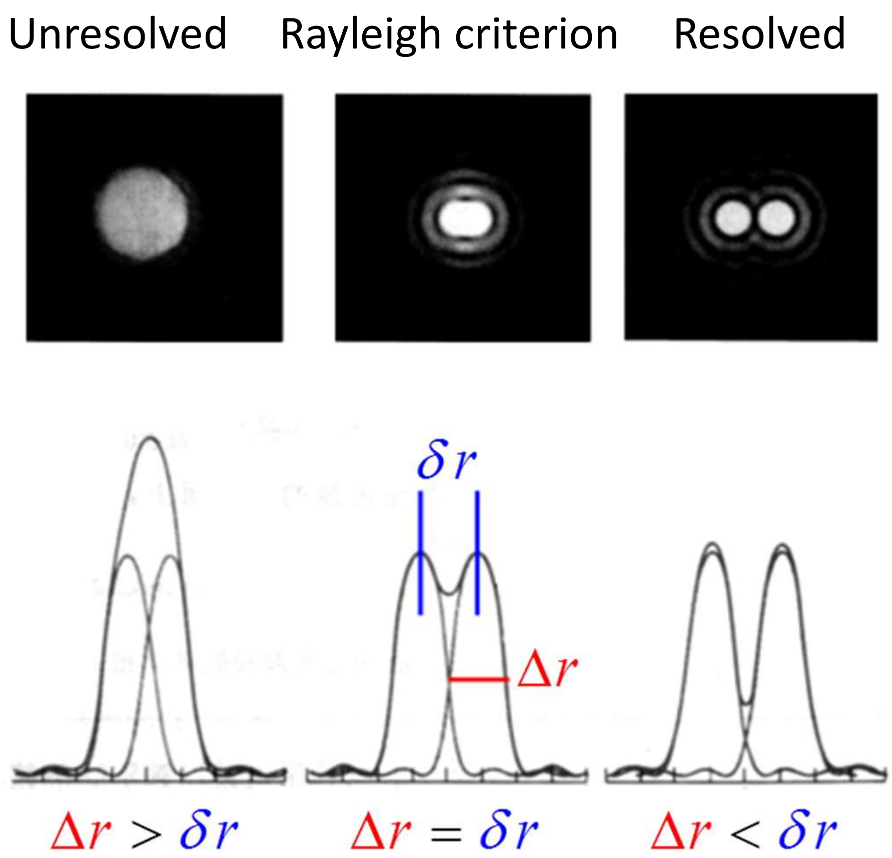
Rayleigh's criterion relates the size of the central spot to the limit at which two objects can be distinguished:

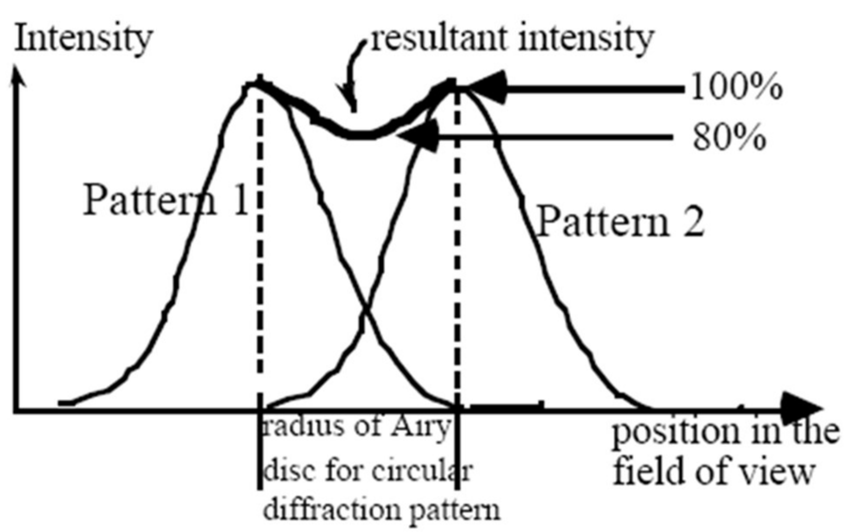
If the first dark fringe of one circular diffraction pattern passes through the center of a second diffraction pattern, the two sources responsible for the patterns will appear to be a single source.

The size of the spot increases with wavelength, and decreases with the size of the aperture.

Resolution-Rayleigh criterion

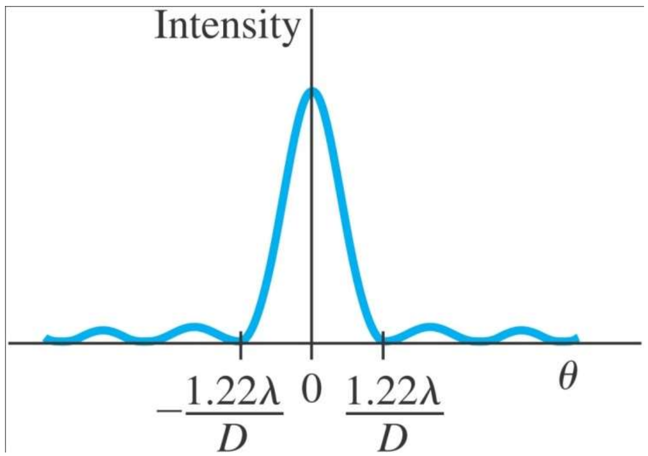
Two wavelength/objects are just resolved when the maximum of one lies at the first minimum of the other.

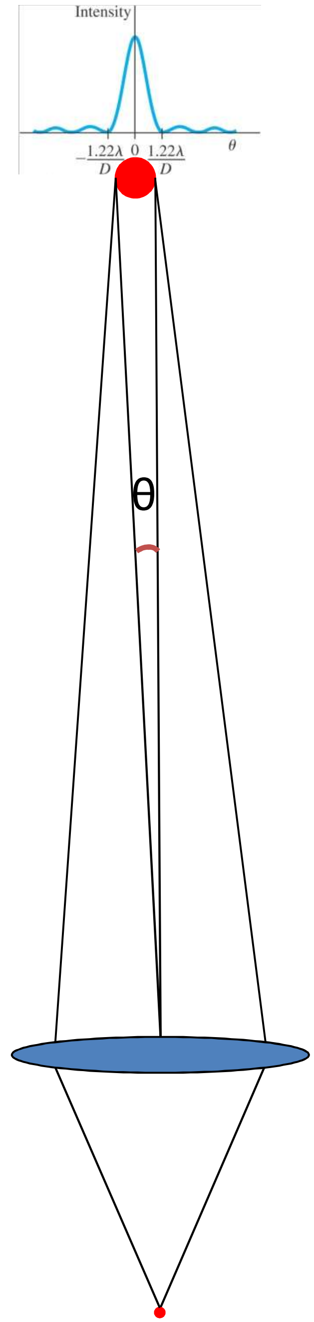




Limits of Resolution; Circular Apertures

When a lens forms the image of a point object, the image in fact is diffraction pattern. For a circular aperture of diameter D, the central maximum has an angular width:

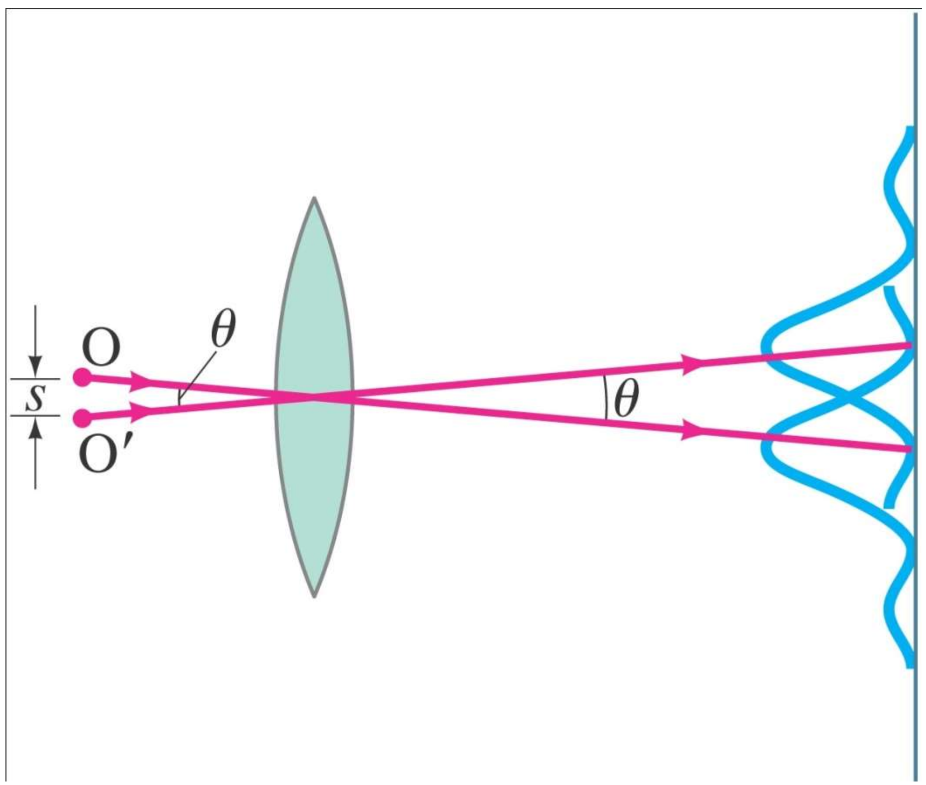




$$\theta = \frac{1.22\lambda}{D} \quad \text{(rad)}$$

Limits of Resolution; Circular Apertures

The Rayleigh criterion states that two images are just resolvable when the center of one peak is over the first minimum of the other.



Example: Hubble Space Telescope.

The Hubble Space Telescope (HST) is a reflecting telescope that was placed in orbit above the Earth's atmosphere, so its resolution would not be limited by turbulence in the atmosphere. Its objective diameter is 2.4 m. For visible light, say λ = 550 nm, estimate the improvement in resolution the Hubble offers over Earth-bound telescopes, which are limited in resolution by movement of the Earth's atmosphere to about half an arc second. (Each degree is divided into 60 minutes each containing 60 seconds, so 1° = 3600 arc seconds.)

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5.50 \times 10^{-7}}{2.4} = 2.8 \times 10^{-7}\,(\text{rad}) = 5.77 \times 10^{-2}\ (\text{arc sec})$$

Almost 10 times better.

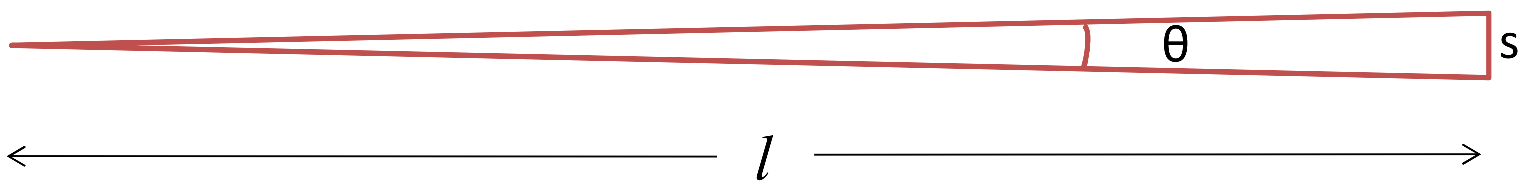
Example: Eye resolution.

You are in an airplane at an altitude of 10,000 m. If you look down at the ground, estimate the minimum separation s between objects that you could distinguish. Could you count cars in a parking lot? Consider only diffraction, and assume your pupil is about 3.0 mm in diameter and λ = 550 nm.

Eye's resolution:

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5.50 \times 10^{-7}}{3.0 \times 10^{-3}} = 2.24 \times 10^{-4}\,(\text{rad})$$

Distinguishable separation s:



$$s = l\theta = 10000 \times 2.24 \times 10^{-4} = 2.24\ \text{m}$$

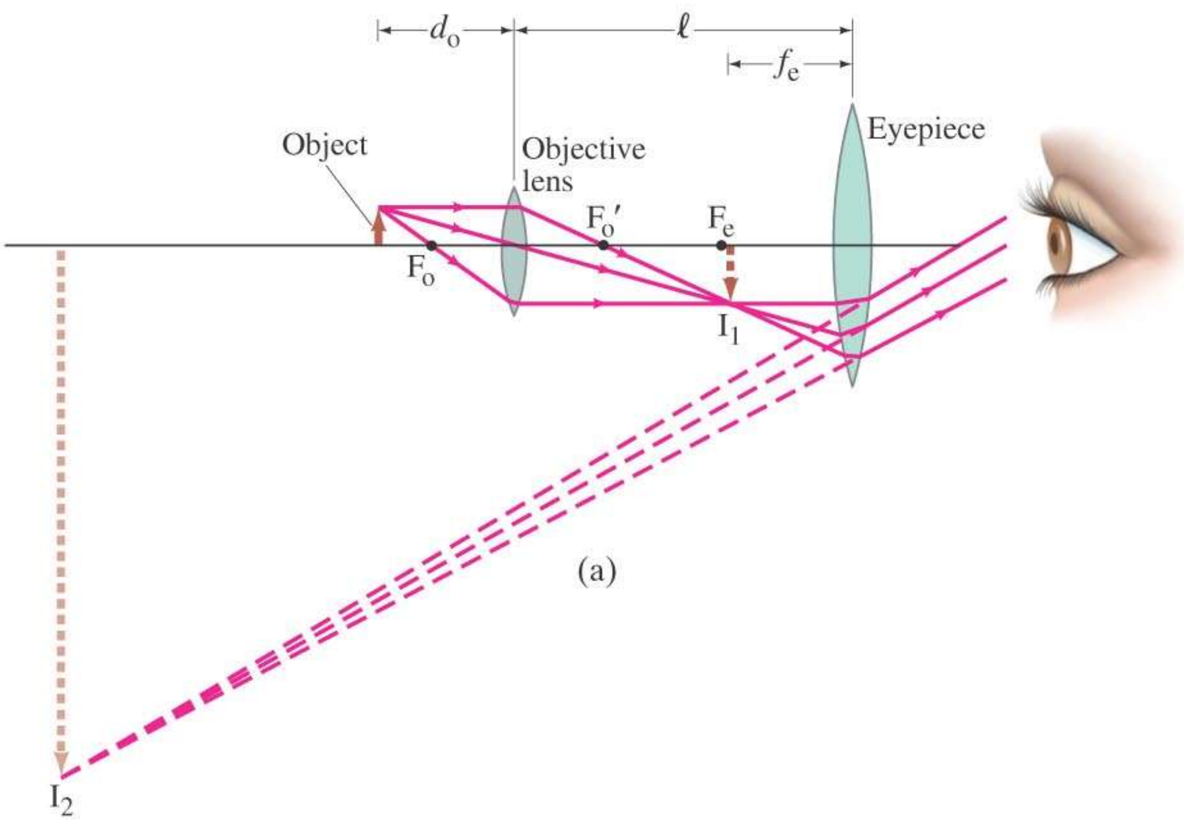
That's about the size of a car.

Resolution of Microscopes; the λ Limit

For microscopes, assuming the object is at the focal point, the resolving power is given by

$$\mathrm{RP} = s = f\theta = \frac{1.22\lambda f}{D}.$$

$$\theta = \frac{1.22\lambda}{D} \quad \text{(rad)}$$



(a)

(b)

Resolution Microscopes; the λ Limit

Typically, the focal length of a microscope lens is half its diameter, which shows that *it is not possible to resolve details smaller than the wavelength being used:*

$$\mathrm{RP} \approx \frac{\lambda}{2}.$$

Resolution of the Human Eye and Useful Magnification

The human eye can resolve objects that are about 1 cm apart at a distance of 20 m, or 0.1 mm apart at the near point.

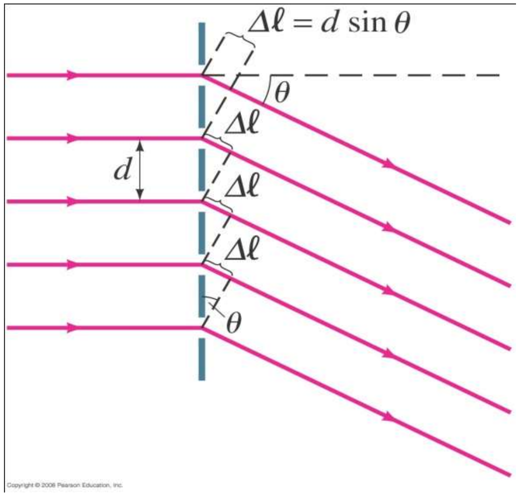
This limits the useful magnification of a light microscope to about 500x–1000x.

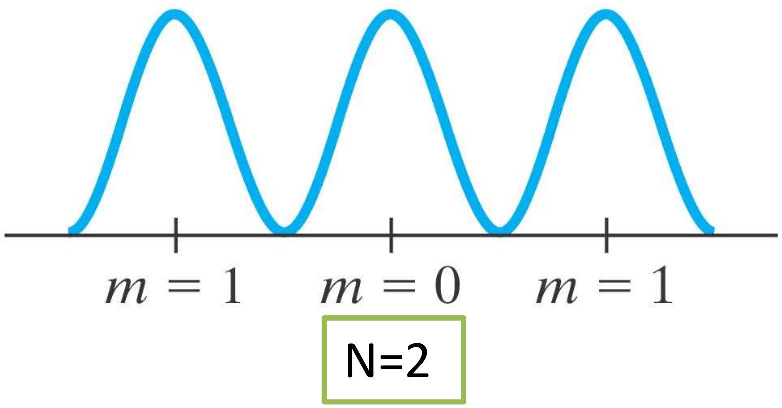
$$\frac{\lambda}{2} \approx 250\text{nm} \approx 2.5 \times 10^{-7} m$$

$$500 \times \frac{\lambda}{2} \approx 1.25 \times 10^{-4} m \approx 0.1 mm$$

Diffraction Grating

A diffraction grating consists of a large number (N) of equally spaced narrow slits or lines. A transmission grating has slits, while a reflection grating has lines that reflect light.

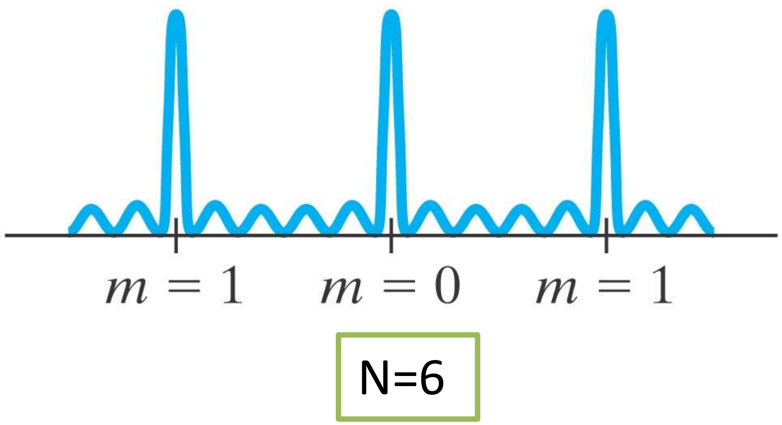




The more lines or slits there are, the narrower the peaks. $I_0 \propto N^2$.

Principal maxima (θ can be large):

$$\sin\theta = \frac{m\lambda}{d}, \qquad m = 0, 1, 2, \cdots$$



Example: Diffraction grating: lines.

Determine the angular positions of the first- and second-order maxima for light of wavelength 400 nm and 700 nm incident on a grating containing 10,000 lines/cm.

$$d = \frac{1}{10000} = 1\times10^{-4}\,cm = 1.0\times10^{-6}\,m$$

The first-order maximum:

$$\sin\theta_{400} = \frac{\lambda}{d} = \frac{4.0\times10^{-7}}{1.0\times10^{-6}} = 0.4, \quad \theta_{400} = 23.6°$$

$$\sin\theta_{700} = \frac{\lambda}{d} = \frac{4.0\times10^{-7}}{1.0\times10^{-6}} = 0.7, \quad \theta_{700} = 44.4°$$

The second-order maximum:

$$\sin\theta_{400} = \frac{2\lambda}{d} = \frac{2\times4.0\times10^{-7}}{1.0\times10^{-6}} = 0.8, \quad \theta_{400} = 53.1°$$

$$\sin\theta_{700} = \frac{2\lambda}{d} = \frac{2\times7.0\times10^{-7}}{1.0\times10^{-6}} = 1.4, \quad \text{No second - order maximum.}$$